MATHEMATICS EDUCATION AND THE iPOD TOUCH

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Apple's iPod Touch is a personal digital device, essentially a version of the popular iPhone, but without telephone capabilities. Although not designed expressly for education, software has been developed for the device for mathematical and educational purposes, while some of its other capabilities (such as those for podcasts, videos and the Internet) can be used for mathematics education. This paper provides an analysis and evaluation of some of these various opportunities for 21st century mathematics education. While some elements of the iPod Touch offer attractions for mathematics education, some educational limitations are also identified.

Introduction

This paper explores a close relative of Apple's popular iPhone, the iPod Touch (here abbreviated simply to iPod), which has been used in some educational settings because of its significant digital capabilities. The most critical differences between the iPod being considered here and the iPhone is that the former has neither telephonic capabilities nor digital camera capabilities In an age in which digital technologies abound, it is appropriate to consider the potentials of a device of this kind for teaching and learning mathematics. Indeed, recent national curriculum initiatives, (Australian Curriculum, Assessment and Reporting Authority, 2011) are designed with modern digital technologies in mind, at least in part. Successive (annual) generations of iPods have become more sophisticated and powerful, as frequently happens with new technologies; this is quite problematic in a paper of this kind, which does not thus claim to be entirely up to date when published.

A focus on the iPod is not intended to suggest a preference for this particular product over others, but is made partly on financial grounds. Apple's more recent iPad series of devices have also created a good deal of excitement and commercial interest, but are presently much too expensive for typical schools.

Applications

From the perspective of mathematics education, the most likely way in which an iPod might be of value is through the use of applications, commonly abbreviated to apps, developed especially for mathematical purposes. Apps are available via Apple's free

software, *iTunes*, which links directly to their iTunes Store. Details are available at Apple Corporation (2011). Apps can be downloaded via computer from the iTunes store or can be downloaded directly to an iPod via the Internet. While some apps are free, others must be purchased, usually for relatively small prices. (At present, the most common purchase price is $1.19, although a few are more expensive.) It is necessary to have an account to download apps, whether or not the apps are free. Updates are free.

There is a very large number of apps available in the App Store, with various classifications used to organise them. For example, apps are classified into categories, including Education, Productivity, Games, Utilities, Reference (in all of which I have found some interesting mathematical examples). A search engine allows for an app to be searched for by name (so finer-grained reference details of the apps mentioned in this paper are not provided), and other searches will allow a number of apps to be identified. The search engine is not the friendliest, so that it can sometimes be hard to navigate all the results and explore all the choices efficiently. For example, when 'math' was used as a search term recently by the author, a little under 6000 apps for an iPod were identified, including examples classified in each of the above categories (as well as some others), with both free and paid apps. Consequently, a paper of this length does not claim to cover the territory exhaustively, but rather intends to provide a perspective on some of the possibilities presently available, with a few examples chosen to illustrate these. There are reviews available online for many apps, especially those that have been around for a while, although it is problematic to place too much reliance on these, with educational interests in mind, without a sense of who the reviewers are.

Many of the free apps (but not all) are in fact reduced or slightly disabled versions of paid apps (and hence are often described as 'lite' versions, increasingly commonly but no more grammatically correctly), offered to provide potential customers with sufficient experience of the approach taken to encourage them to purchase the paid version, sometimes even with irritating and frequent messages in the form of advertisements to do so. This is of course understandable, as those producing the apps rely on their sales to support their businesses. While data are not available (to the author, at least), it is easy to get the impression that many of the app developers do not have much mathematics educational background or expertise, so that what is offered is not always pedagogically sound or even mathematically interesting. In addition, with the recent emergence of the iPad and iPad2, a good deal of energy is directed at making new versions of iPod apps to run on the new devices, or making apps only for the iPad, not the older and less sophisticated iPod, not a surprise for commercial organisations.

What follows is a brief and unavoidably personal account of some of the kinds of offerings presently available, especially in relation to school mathematics, mostly with an eye to material likely to be of interest to secondary schools. As noted earlier, no claim to exhaust the territory is provided here.

Graphing

Many apps allow users to graph and explore functions and other sorts of graphs. While secondary students might usually have access to a graphics calculator for this purpose, some of these apps offer some nice features that allow for easy manipulation of both 2D and 3D graphs, and using more colours and a higher resolution than a typical graphics calculator. As the iPod is operated mostly by finger movements on a touch-sensitive

screen, this adds a new sort of experience including moving (two or three) axes around, zooming in and out by stretching the coordinate plane or 3D space with two fingers at once and locating points of intersection by just touching them. Figure 1 shows two good examples, the first from *GraphCalc* and the second from *Quick Graph*.

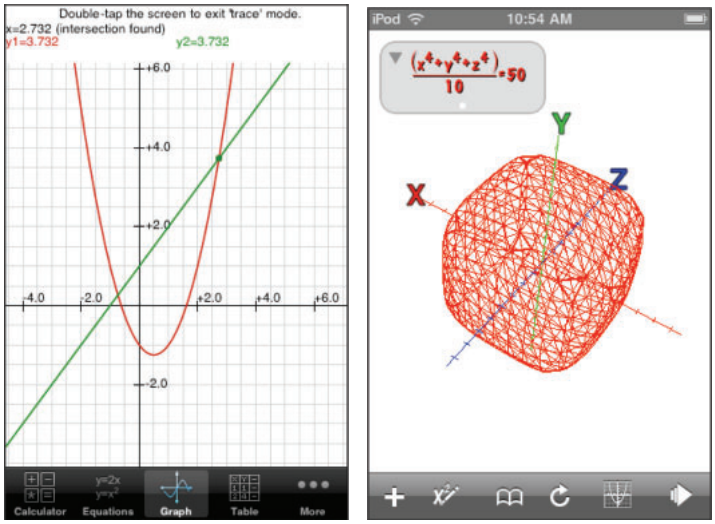

Figure 1. GraphCalc and Quick Graph, two examples of graphing apps.

With apps of these kinds, students might reasonably be expected to get a different and even sensory experience of graphing functions than a graphics calculator can offer. For example, many teachers have referred to tracing as being like moving one's finger along a graph; with a touch-screen of this kind, this is precisely what a user does.

Figure 2. Two other graphing apps, GraphBook and iTrig.

As well as providing graphical capabilities that in some ways match what students' graphics calculators might provide, some apps offer different graphing experiences. For example, *SpaceTime* is a recent and very expensive app ($23.99 at present, around the same price as a scientific calculator, but much more than typical apps) that claims to provide high-end and programmable features such as some those involved in *Mathematica* and *MatLab*. Figure 2 shows the *GraphBook* app, which seems to have

been constructed to provide some animated and manipulable examples of the *SpaceTime* capabilities, as an inducement to purchase the complete app. Nor are graphical apps restricted to graphing functions. Figure 2 shows a screen dump from the *iTrig* app, which provides a unit circle around which a point can be moved with one's finger and associated graphs and values are shown.

These examples do not exhaust the possibilities; many other apps have a graphical element. For example, *4D Spin* addresses the nature of the fourth dimension, *Polar Sweep* is concerned with relationships between rectangular and polar coordinates and many apps such as *Fractals* allow students to explore fractal images of various kinds.

Calculator

A very large number of apps offer a calculator of some kind, and the standard iPod even comes with a calculator app. There are many kinds of specific calculators. An example is the right triangle solver in *iTrig*, to calculate lengths of sides and angles of a right triangle from partial information. The standard iPod calculator is both an arithmetic and scientific calculator (although some users may not realise this unless they turn or shake their iPod). The scientific calculator displays more places of decimals than a standard scientific calculator and in that sense is an improvement, as shown in Figure 3.

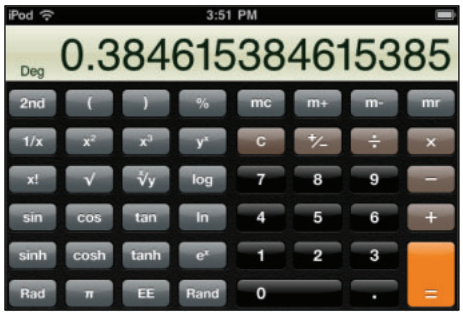

Figure 3. The standard iPod Calculator app in scientific mode.

There are iPod apps for just about any kind of calculation likely to be needed in secondary school, including unusual tasks such as those shown in Figure 4.

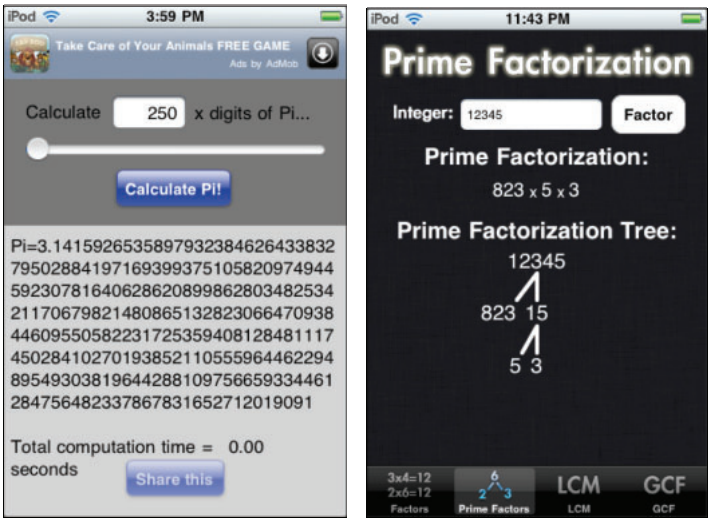

Figure 4. Calculators are involved in both iFactorization and Pi apps.

Others include algebraic calculations with *PocketCAS*, normal distribution tables with *GaussPad*, and unit conversions with *Units*. There are even old-style calculators such as an *Abacus*, *iSlideRule* and *Longhand Division*, each of which operates successfully. At least collectively, the large suite of calculator apps available will make it clear to students that many mathematical calculations can be automated for machines to do, and that they need to continue to choose the right tool for the job (which includes mental and approximate calculations some times, of course).

Reference source

A surprising number of apps seem to function as mathematical reference works, with tables of formulae, diagrams, theorems, and other items. It can be quite useful to have references of these kinds available when needed, especially the slightly more esoteric ones (which of course varies from person to person). Apps like these might reflect an image of those who have constructed them of mathematics as a discipline in which there are many formulae to memorise (or look up). Figure 5 shows three typical examples.

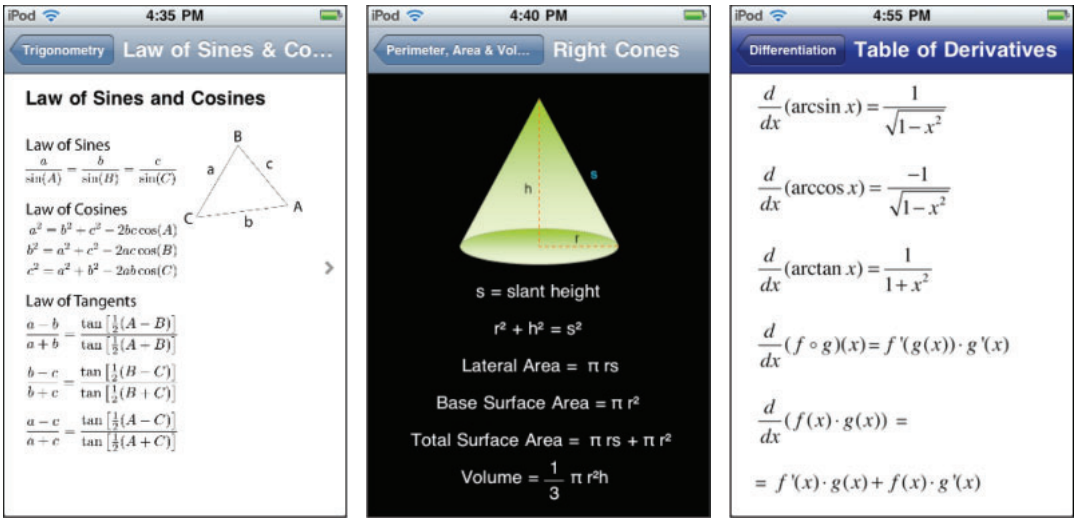

Figure 5. Pages from Math Reference Free, Math Pro and Formulas apps.

Measuring

A number of apps have been designed to handle various measurement tasks, often found in mathematics, although it is questionable whether an iPod version of these is a superior tool to the original measuring tool.

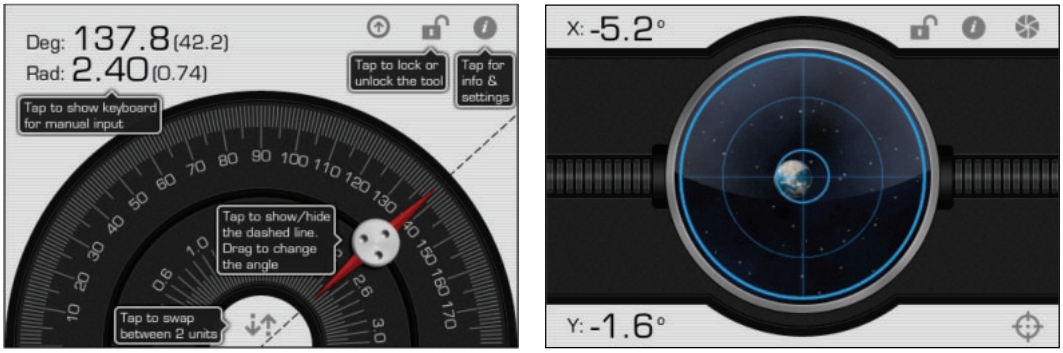

Figure 6. Protractor Deluxe and Measures apps seem to offer spurious accuracy.

Indeed, it seems that there are examples here of computer programmers making something in order to prove that it can be done, rather than to produce a genuinely useful tool. Figure 6 shows two of many possible examples. The protractor, surprisingly, seems to measure angles only in a clockwise rather than anticlockwise direction and to display an accuracy that is substantially beyond what is reasonable with the actual device in practice. Similarly, the spirit level (one of a suite of tools in the same app), in measuring a surface to the nearest tenth of a degree seems to overstate the accuracy of the measurement of extent to which my desk is horizontal. Other examples, such as *RulerPlus*, *Tape Measure* and *iHandy Carpenter*, can be similarly criticised.

Drill and practice

At first glance, the available apps for mathematics or for education seem to suggest that the most useful tool for iPods involves lots of practice of mathematical skills, especially those related to computation, with many of them focussed on the primary years of schooling. Practice certainly has an important place in school mathematics, and a device that uses colour, entertainment and novelty effects to engage students in practice at a range of levels may be a useful supplement to other experiences. Despite enthusiastic claims to the contrary by the designers, many of the apps I examined in this category seem to offer not much more than heaps of practice, often timed and speeded and generally with feedback; overall there seems to be a limited case to use an expensive piece of digital technology in such a mundane way. Indeed, Pelton (2011) suggested that around 40% of the designated 'top' apps for mathematics fell into the category of drilling basic facts. Some of these apps are designed in the form of flash cards, some as games and others differently, but fundamentally many offer little of lasting conceptual value, and it is questionable that students would be attracted to them for very long, once the novelty had worn off.

Some apps that essentially provide a form of practice do so in a slightly more engaging and interesting way, however. Figure 7 shows two examples, *Motion Math* and *Number Line*. The *Motion Math* app makes use of the motion sensor devices that are an integral part of the iPod and iPhone®, so that the device knows when and how it has been turned.

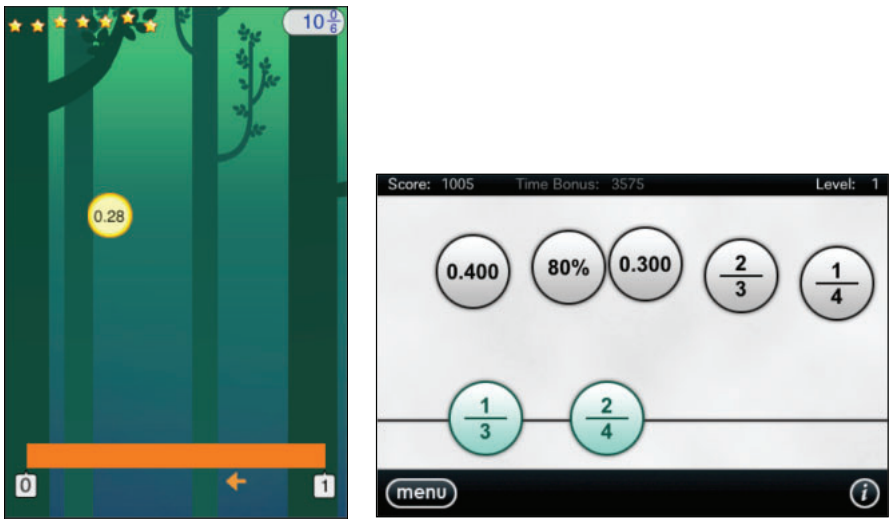

Figure 7. Motion Math and Number Line.

The screen shows a bouncing ball with a fraction or decimal inside. The user must tilt the device to make the ball land on (or close to) the appropriate point at the bottom between 0 and 1, in order to get a new number and to try again.

Similarly, *Number Line* requires the user to drag the numbered balls onto the number line in the correct numerical order. Each of these games is fundamentally concerned with understanding and comparing the sizes of numbers, in different representations, and seems to exploit better the educational possibilities than do many of the other apps which seem merely to automate what could as easily have been placed on a worksheet.

Miscellaneous

There are many other kinds of apps that might find a place in secondary school mathematics, and even be of interest to teachers themselves - too many to easily classify. Figure 8 shows two examples.

Figure 8. Samples from the MathFunFacts and Is That Prime? apps.

The *MathFunFacts* app, created by Francis Su of Harvey Mudd College in USA has a large number of mathematical snippets, many with fairly recent mathematics, that teachers may find of interest. *Is That Prime?* provides quick information about the primality (or the factors) of integers. While such apps might be regarded as a little quirky, they may still find a place in a modern classroom.

Figure 9 shows three quite different examples, each of which may have some kind of appeal to secondary students, while being out of the mainstream mathematics curriculum. The app, *Discover the Magic of M.C.Escher* contains many of Escher's famous images as well as other written information and activities. *Polyhedron* contains very many images of rotating polyhedra, while *Mathemagics* contains a large collection of number tricks, fertile ground for algebraic thinking.

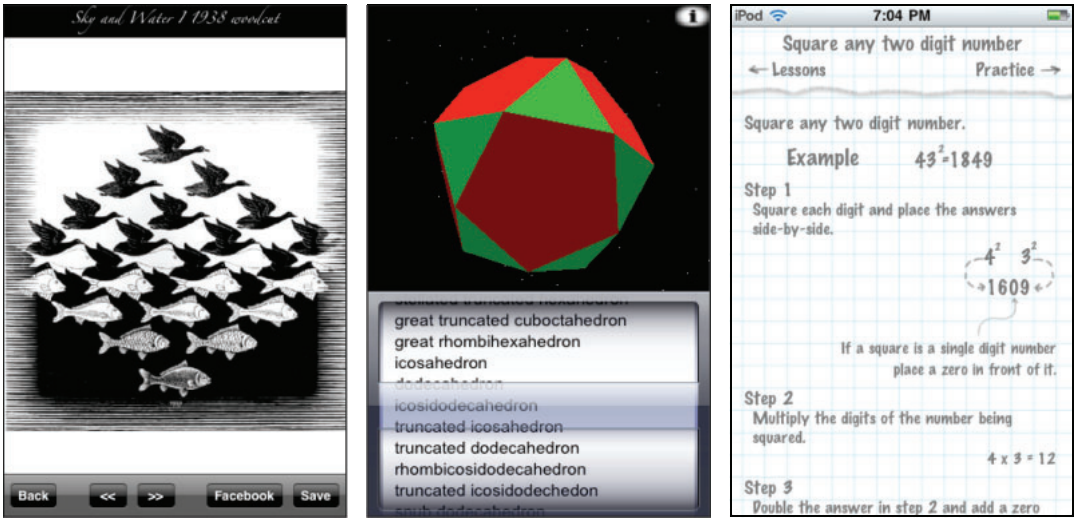

Figure 9. The Magic of Escher, Polyhedron and Mathemagics apps. All M. C. Escher works © 2011 The M.C. Escher Company, the Netherlands. All rights reserved. Used by permission. www.mcescher.com

As well as these, other apps such as *iBooks* provide a mechanism to read electronic books (some of which are free, courtesy of the Gutenburg Project, while others cost close to normal book prices), some apps such as *Chromatics* contain a great deal of visual mathematics for browsing, others such as *Collatz* explore particular bits of mathematics, while yet others, such as Pearson's *Trigonometry*, contain direct instructional materials. There are many other games, puzzles, patterns, spatial and numerical environments that contain elements of mathematics, all too difficult to classify here, but many seem worthy of a second look.

Internet

While applications offer the most likely use of an iPod, the capability of accessing the Internet on a wireless network at school or home leads to other possibilities as well. Kissane (2009) suggested a number of ways in which Internet access might be helpful for mathematics education, and many of these can be used with an iPod. In some cases, the iPod web browser is not needed, as a special app has been constructed for a similar purpose. A good example of this is *WolframAlpha* the very sophisticated search engine with the awesome power of Stephen Wolfram's *Mathematica* behind it, aiming to make all systematic knowledge immediately computable, accessing all available data. Another good example is *Wikipedia*, which has good entries related to mathematics.

A major limitation, however, of Internet use with the iPod is the lack of either Java or Flash capabilities. A consequence of this is that many excellent interactive websites of value for mathematics education (such as the *National Library of Virtual Manipulatives*, the NCTM's *Illuminations* and many parts of the *Nrich* site, as well as interactive software like *GeoGebra*) are rendered inoperative. These limitations might be removed in the future, but it seems that Apple is at present resolutely opposed to these platforms, preferring other approaches to interactive web-based materials. Until a solution to this problem is found, a major potential use of the iPod as a web browser will be severely curtailed, unfortunately.

Podcasts

While many iPod users regard their device as essentially a personal music machine, there are potential uses of the audio and video capabilities for mathematics education as well. Podcasts and video podcasts made locally (for example, by a teacher at a school, or by university staff for external students) or externally (disseminated through the Internet) can carry powerful and interesting mathematical messages. Excellent examples are available via the iTunes site on iTunesU or via the podcasting links on the website. From the UK, the regular *PLUS* podcasts, various Open University series, Marcus Du Sautoy's *A Brief History of Mathematics* from the BBC and the *Travels in a Mathematical World* collection from the Institute of Mathematics and its Applications are all good examples of contemporary mathematical materials that would be of interest to both teachers and older secondary students. The Swede Hans Rosling's *GapCasts* (using the wonderful *GapMinder* software) also offer excellent stimulating materials on an iPod related to the use of statistics to understand modern social and health issues internationally and don't require a live Internet link after downloading.

Projection

A major limitation of the iPod is the present inability of teachers to use it to communicate to a class, through a data projector or large television set. This is a consequence of the design of the device, which is thus only able to be used by one or two students at once. An exception is that some videos and podcasts can be shown on a television set with the appropriate cables. However, the lack of capacity to show apps to a wider audience is a significant educational limitation, which needs to be overcome.
It is possible, but sometimes a little difficult, to use a web camera or other visualisation device to project an iPod screen to a computer and thence to a data projector, but it would be much preferable for there to be a direct link.

Conclusion

While there are some nice apps for the iPod Touch, and some interesting potentials, there are also a lot of uninteresting apps as well as significant practical limitations for the use of the device in mathematics education. As a fairly expensive device (at present), the iPod Touch may be of limited lasting value as a mathematics education learning device, with substantial Internet limitations, while the lack of projection capability is a severe constraint on use for teaching in most cases. Hopefully, refinements to devices of these kinds in the future will address such shortcomings and provide a form of mobile technology that meets the needs of students and schools, and offers significant teaching and learning opportunities.

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